

Vibrations of a Girder on Rigid Supports of Finite Mass Interacting With Soil under Seismic Loads

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Abstract. Transverse vibrations of a single-span girder bridge are considered in the article; the pile part of the bridge interacts with the surrounding soil under seismic action. We assume that the strain of the structure does not go beyond the elastic limit, and the vibrations are linear. The bridge supports are assumed to be immersed in soil and interact with a rigid body under the impact of unsteady dynamic influences. We consider the case when the right and left supports have equal masses and interact with the surrounding soil. Here the symmetry condition is applied, so it is sufficient to consider the equation for the right half of the girder. The problems are solved by the analytical Fourier method under given boundary conditions. The results obtained are analyzed and presented in the form of the distribution of displacements and stresses over the time and length of the bridge structures.

1 Introduction

Various transport structures, including bridge structures, are of great importance worldwide for expanding trunk road networks, increasing the volume of passenger and cargo traffic, and developing the infrastructure of large cities. Bridge structures, being one of the types of construction objects, have specific consumer properties that determine their purpose and quality.

Most of the territory of Uzbekistan is in unfavorable conditions in terms of bridge supports operation. The aggressive impact of the environment and reagents used in the bridge structure operation adversely affect the technical condition and durability of bridge supports on highways.

The most important feature of the road industry is its high social and economic significance; the quality of life of all segments of the population and the development of the economy as a whole depend on the effective functioning of this system.

Over the years of independence, large-scale work has been conducted in the republic to develop road transport infrastructure, ensuring safe interstate transportation and extensive transport links between the administrative centers of regions and districts [1].

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Most of the basic structures in modern capital construction are made of various types of reinforced concrete. It is especially important to accelerate scientific and technological progress in this area at such a gigantic pace in construction. To solve this problem, it is necessary, first, to further develop the methods for calculating and designing reinforced concrete structures by improving the elements of the general theory of strength and deformability of concrete and reinforced concrete, based on their real properties and performance under various operational and seismic (strong and weak) impacts including the states close to destruction.

Analysis of data on seismic damage showed that the impact of earthquakes of magnitude 7-9 on roads built according to normal standards leads to substantial damage to structures and serious disruptions to traffic, up to a complete cessation of traffic for a period of several days to several weeks. The failure of bridges during a possible earthquake can lead not only to the costs of restoration or construction of a new structure. The lack of transport access in emergency cases may complicate the work of rescuers and may lead to increased loss of human lives due to the delayed response [1-3].

2 Methods

Structure vibrations during the earthquakes induced by the oscillatory motion of the base are called seismic vibrations. Seismic vibrations of structures are of a very complex spatial nature. Under intense seismic impact, leading to damage, the strains in the structure go beyond the elastic limits, and vibrations, as a rule, are not linear. However, to simplify the problem, the normalized method for determining seismic forces based on linear theory allows independent consideration of three mutually perpendicular vibration components (vertical and horizontal ones).

During earthquakes, the supports and spans of bridges dynamically interact with each other, and their complex reacts to the foundation motion as a single oscillatory system. Therefore, the main task of the theory of seismic vibrations of bridge structures is to study the joint vibrations of spans and supports caused by vibrations of the foundation. At present, this problem does not have a complete solution [2-11].

In the applied dynamics of structures, bridges are one of the main objects of research. However, these studies are mostly related to the dynamic effects of rolling stock and consider, as a rule, vertical (or spatial) vibrations of spans, regardless of the supports. The analysis of joint vibrations of these elements is rare [2], [12-15].

Bridge supports can experience both displacements across the bridge's axis caused by bending and shear strains of their structures and the foundation flexibility and torsional strains (section rotation in the horizontal plane). Obviously, torsional strains occur only in rare cases and are not significant [2].

Based on the above, in the study of seismic vibrations of girder bridges in the first approximation, we can limit ourselves to considering only the transverse strains of spans and supports. This is all the more acceptable since transverse strains play a determinant role in the formation of horizontal seismic forces.

This article aims to study the transverse vibrations of a single-span girder bridge, the pile part of which interacts with the surrounding soil under seismic action Academician T. Rashidov has developed a dynamic theory of seismic resistance of complex systems of underground structures based on considering the difference in strains of a structure and soil [16-17]. Several scientists in our country and abroad have researched the problems of seismic resistance of underground and surface structures interacting with surrounding soil [18-23].

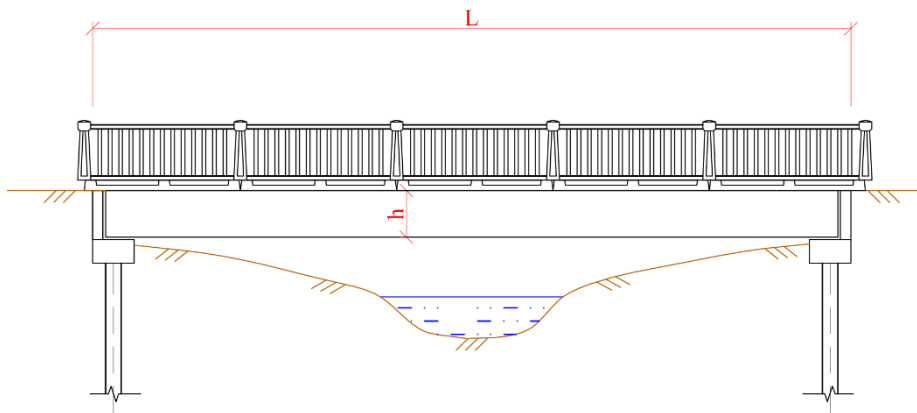


Fig. 1. Scheme of a single-span girder road bridge

To assess the reliability and bearing capacity of road girder bridges on supports, in addition to the moving loads acting on the girder, we should take into account the forces transmitted through the girder supports related to the impact of, for example, seismic waves. Consider a road girder with two end supports (figure 1). As a first approximation, the bridge supports are assumed to be immersed in the ground and interacting with a rigid body under the influence of nonstationary dynamic influences. The origin of coordinates is set at point A , the Ox axis is directed along the neutral axis of the girder, and the axes Oy_1 (with the origin at point O_1) are perpendicular to it (figure 2). Let a longitudinal wave flow around the supports, behind the front of which the soil particles' motion depends on the coordinate y_1 and time t according to the law $u_0 = u_0(t - y_1/c_0)$. The presence of supports in the boundary sections of the girder leads to the emergence of concentrated forces, which can be taken into account through the discontinuities of the third derivative in the equation of motion. Under these assumptions, the girder deflection $y = y(x, t)$ satisfies the following equation

We set the origin at point A , direct the Ox axis along the neutral axis of the girder, and the Oy_1 axes (with the origin at point O_1) are perpendicular to it (figure 2)

$$m_B \frac{\partial^2 y(x, t)}{\partial t^2} + EJ_z \frac{\partial^4 y(x, t)}{\partial x^4} = 0 \tag{1}$$

and the following zero initial and boundary conditions

$$\frac{\partial y}{\partial x} = 0, EJ_z \frac{\partial^3 y}{\partial x^3} = -M_1 \frac{\partial^2 y}{\partial t^2} - k_{01}[y(0, t) - u_0(t)] - k_{11}[y(0, t) - u_1(t)] \text{ for } x = 0 \tag{2}$$

$$\frac{\partial y}{\partial x} = 0, EJ_z \frac{\partial^3 y}{\partial x^3} = M_2 \frac{\partial^2 y}{\partial t^2} + k_{02}[y(l, t) - u_0(t)] + k_{12}[y(l, t) - u_1(t)] \text{ for } x = l \tag{3}$$

where m_B is the linear weight of the girder, E is the Young's modulus of the material of the girder, J_z is the moment of inertia of the section, l is the length of the girder, M_1 and M_2 are the weights of the left and right support, and k_{01} and k_{02} are the stiffness coefficients of the left and right support in soil. u_0 is the soil particles' motion behind the front of the incident

longitudinal wave, k_{11} and k_{12} are the longitudinal shear coefficients on the contact surface of the supports with soil $u_1 = \frac{1}{H} \int_0^{c_0 t} u_0(t - y_1 / c_0) dy_1$ for $t < H / c_0$,
 $u_1 = \frac{1}{H} \int_0^H u_0(t - y_1 / c_0) dy_1$ for $t > H / c_0$, H – is the length of supports.

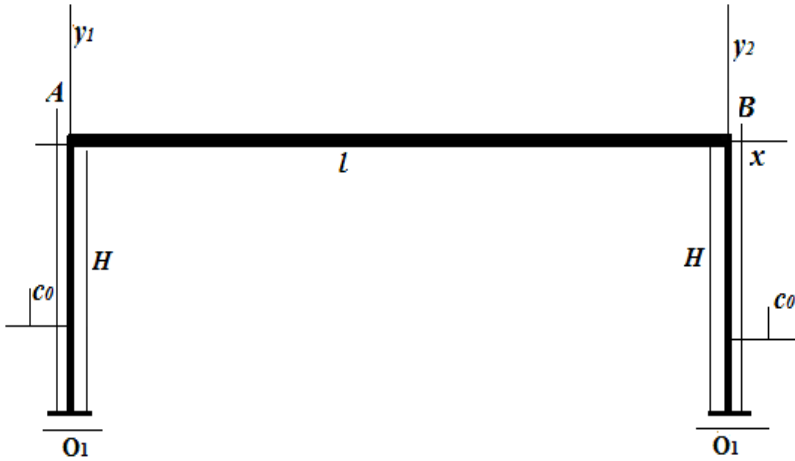


Fig. 2. Design scheme of a road girder bridge with movable supports

Consider the case when the right and left supports have equal weights and interact with the surrounding soil. We assume that $M_1=M_2=M$, $k_{01}=k_{02}$, and $k_{02}=k_{12}$, and using the symmetry condition, it is sufficient to consider the equation for the right half of the girder

$$m_B \frac{\partial^2 y(x,t)}{\partial t^2} + EJ_z \frac{\partial^4 y(x,t)}{\partial x^4} = 0 \tag{5}$$

and to require the fulfillment of the symmetry condition in the middle section of the girder.

$$\frac{\partial y}{\partial x} = 0, \quad EJ_z \frac{\partial^3 y}{\partial x^3} = -M \frac{\partial^2 y}{\partial t^2} - k_{01}[y(0,t) - u_0(t)] - k_{11}[y(0,t) - u_1(t)] \text{ for } x = 0 \tag{6}$$

$$\frac{\partial y}{\partial x} = 0, \quad \frac{\partial^3 y}{\partial x^3} = 0 \text{ for } L = l/2.$$

Assuming that $\xi = x / L$, we introduce a new function by the following formula

$$y = \bar{y} + A(t)L\xi^2(\xi^2 - 4\xi + 4)$$

where the function $\bar{y}(\xi, t)$ satisfies the following conditions

$$\frac{\partial \bar{y}}{\partial \xi} = 0 \text{ for } \xi = 0 \tag{7}$$

$$\frac{\partial \bar{y}}{\partial \xi} = 0, \frac{\partial^3 \bar{y}}{\partial \xi^3} = 0 \text{ for } \xi = 1 \tag{8}$$

The second condition in (6) can be reduced to a homogeneous form if the function $A(t)$ is selected by the following formula

$$A = -\frac{L^2 [k_{01}u_0(t) + k_{11}u_{11}(t)]}{24EJ_z}$$

Then the second boundary condition (6) with respect to the function $\bar{y}(\xi, t)$ is written in the following form

$$\frac{EJ_z}{L^3} \frac{\partial^3 \bar{y}}{\partial \xi^3} = -M \frac{\partial^2 \bar{y}}{\partial t^2} - (k_{01} + k_{11})\bar{y}(0, t) \text{ for } \xi = 0 \tag{9}$$

In this case, the equation of motion of the girder relative to $\bar{y}(\xi, t)$ is written in the following form

$$m_B \frac{\partial^2 \bar{y}(\xi, t)}{\partial t^2} + \frac{EJ_z}{L^4} \frac{\partial^4 \bar{y}(x, t)}{\partial \xi^4} = -m_B A''(t) L f_0(\xi) - A(t) \frac{EJ_z}{L^3} f_0^{IV}(\xi), \tag{10}$$

where $f_0 = \xi^2 (\xi^2 - 4\xi + 4)$.

The solution of equation (10) under boundary conditions (7) – (9) can be obtained by the Fourier method, following which the solution of the homogeneous equation corresponding to (9) can be represented in the following form

$$\bar{y} = \varphi(\xi)T(t)$$

we assume that $\ddot{T} = -\omega^2 T(t)$. The function $\varphi(\xi)$ satisfies the equation

$$\varphi^{IV} - \lambda^4 \varphi = 0 \tag{11}$$

And boundary conditions

$$\varphi' = 0, \varphi''' = (\lambda^4 \bar{M} - \beta)\varphi \text{ for } \xi = 0 \tag{12}$$

$$\varphi' = 0, \varphi''' = 0 \text{ for } \xi = 1 \tag{13}$$

where $\lambda = L^4 \sqrt{\frac{m_B \omega^2}{EJ_z}}$, $\beta = \frac{(k_{01} + k_{11})L^3}{EJ_z}$, $\bar{M} = M / m_B L$.

The solution of equation (12) is presented in terms of the Krylov functions (C_i are arbitrary constants)

$$\varphi = C_1 Y_1(\lambda \xi) + C_2 Y_2(\lambda \xi) + C_3 Y_3(\lambda \xi) + C_4 Y_4(\lambda \xi), \quad 0 < \xi < 1 \quad (14)$$

where $Y_i(z)$ are the Krylov functions

$$Y_1(z) = (chz + \cos z) / 2, \quad Y_2(z) = (shz + \sin z) / 2, \quad Y_3(z) = (chz - \cos z) / 2,$$

$$Y_4(z) = (shz - \sin z) / 2.$$

Conditions (12), (13) imply

$$C_2 = 0, \quad C_4 = \frac{\lambda^4 \bar{M} - \beta}{\lambda^3} C_1, \quad (15)$$

$$C_1 [Y_4(\lambda) + \frac{\lambda^4 \bar{M} - \beta}{\lambda^3} Y_3(\lambda)] + C_3 Y_2(\lambda) = 0, \quad (16)$$

$$C_1 [Y_2(\lambda) + \frac{\lambda^4 \bar{M} - \beta}{\lambda^3} Y_1(\lambda)] + C_3 Y_4(\lambda) = 0. \quad (17)$$

Equating the determinant of the system of equations to zero for C_1 and C_3 , we derive an equation for determining the eigenvalues $\lambda = \lambda_i$

$$\lambda^3 [Y_4^2(\lambda) - Y_2^2(\lambda)] + (\lambda^4 \bar{M} - \beta) [Y_3(\lambda) Y_4(\lambda) - Y_1(\lambda) Y_2(\lambda)] = 0.$$

Setting $C_1=1$, we represent the expressions for the Eigenfunctions $\varphi_i=(\xi)$ in the form

$$\varphi_i = Y_1(\lambda_i \xi) - \frac{\lambda_i^3 Y_2(\lambda_i) + (\lambda_i^4 \bar{M} - \beta) Y_1(\lambda_i)}{\lambda_i^3 Y_4(\lambda_i)} Y_3(\lambda_i \xi) + \frac{\lambda_i^4 \bar{M} - \beta}{\lambda_i^3} Y_4(\lambda_i \xi).$$

It can be shown that the Eigenfunctions $\varphi_i=(\xi)$ satisfy the generalized orthogonality condition

$$m_b \int_0^1 \varphi_i \varphi_k d\xi + M \varphi_i(0) \varphi_k(0) / L = 0 \quad \text{for } i \neq k. \quad (18)$$

The solution to equation (10) is represented as the sum

$$\bar{y} = \sum_{n=1}^{\infty} \varphi_n(\xi) T_n(t). \quad (19)$$

Substituting expression (19) into equation (10), we obtain

$$\sum_{n=1}^{\infty} m_b (\ddot{T}_n + \omega_n^2 T_n) \varphi_n(\xi) = -m_b A'' L f_0(\xi) - \frac{EJ_z}{L^3} A f_0^{IV}(\xi)$$

Using the orthogonality condition (18), we compose an equation for the expansion coefficients

$$\ddot{T}_n + \omega_n^2 T_n = F(t) = -\frac{1}{\|\varphi_n\|} \int_0^1 [LA''f_0 + A \frac{EJ_z}{m_B L^3} f_0^{IY}] \varphi_n d\xi$$

where $\|\varphi_n\| = m_n \int_0^1 \varphi_n^2 d\xi + M\varphi_n^2(0) / L$.

Solution of the last equation under initial conditions

$$T_n(0) = 0, \dot{T}_n(0) = \frac{m_n L \dot{A}(0)}{\|\varphi_n\|} \int_0^1 f_0(\xi) \varphi_n(\xi) d\xi$$

has the form

$$T_n = \frac{1}{\omega_n} \int_0^t F_0(\tau) \sin \omega_n(t - \tau) d\tau + \frac{\dot{T}_n(0)}{\omega_n} \sin \omega_n t$$

If the change in the displacement of soil particles behind the wave front is taken according to the law $u_0 = U_0 \sin \frac{c_0 t - y_1}{L_v}$ (L_v is the wavelength, U_0 is the maximum displacement of soil particles behind the wave front), then we have

$$A = -U_0 \frac{L^2}{24EJ_z} \left(k_{01} \sin \frac{c_0 t}{L_v} + k_{11} \frac{L_v}{H} (1 - \cos \frac{c_0 t}{L_v}) \right) \text{ for } 0 < t \leq H / c_0$$

$$A = -U_0 \frac{L^2}{24EJ_z} \left(k_{01} \sin \frac{c_0 t}{L_v} + k_{11} \frac{L_v}{H} (\cos \frac{c_0 t - H}{L_v} - \cos \frac{c_0 t}{L_v}) \right) \text{ for } t \geq H / c_0.$$

3 Results and Discussion

Figures 3, 4 show the curves of the dependence of deflection y (figure 3) and bending stresses $\sigma = EJ \frac{\partial^2 y}{\partial x^2} / W$ (W – is the moment of resistance of the girder section) (figure 4) in different sections in time t (sec) for a girder of a rectangular cross-section of width $b=0.3$ m and of height $h=0.6$ m under the action of wave $u_0 = U_0 \sin \frac{c_0 t - y_1}{L_v}$.

The calculations were conducted for two wavelengths L_v and taken as $U_0=0.005$ m, $c_0=1000$ m/s, $k_{01}=1.7 \cdot 10^5$ N/m, $k_{11}=0.2k_{01}$, $E=5 \cdot 10^{10}$ Pa, $L=25$ m, $M=2500$ kg, $m_B=100$ kg/m, $H=8$ m.

Analysis of the graphs shows that the action of a harmonic wave on a girder through rigid supports leads to an oscillatory law of change in deflections and stresses in its sections. In this case, the maximum values of stresses are observed in the sections of the

girder attached to the supports, and their values significantly decrease with distance. This indicates the possibility of the appearance of plastic strains near these sections, which is why the decrease in the quality of the operational parameters of the girder bridge. A decrease in the wavelength increases the frequencies and practically does not affect the amplitude of the girder deflections.

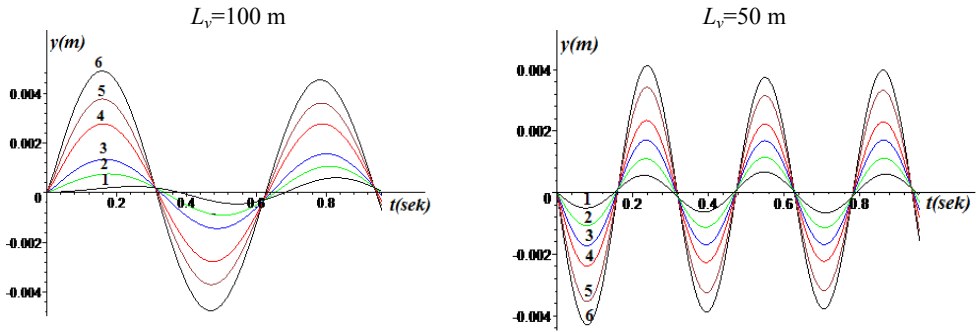


Fig. 3. Change in deflection in different sections of the girder $x(m)$ (referred to L) in time t (sec) for two values of wavelength $L_v(m)$: 1 – $x/L = 0.2$, 2 – $x/L = 0.3$, 3 – $x/L = 0.4$, 4 – $x/L = 0.5$, 5 – $x/L = 0.7$, 6 – $x/L = 1$

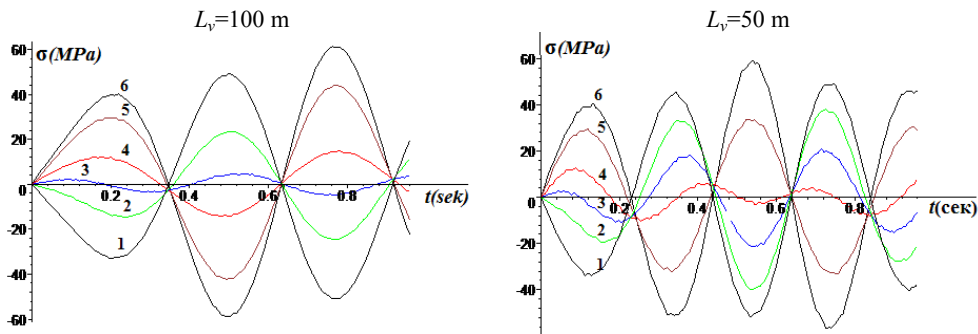


Fig. 4. Change in bending stresses σ (MPa) in different sections of the girder $x(m)$ (referred to L) in time t (sec) for two values of wavelength $L_v(m)$: 1 – $x/L = 0.2$, 2 – $x/L = 0.3$, 3 – $x/L = 0.4$, 4 – $x/L = 0.5$, 5 – $x/L = 0.7$, 6 – $x/L = 1$

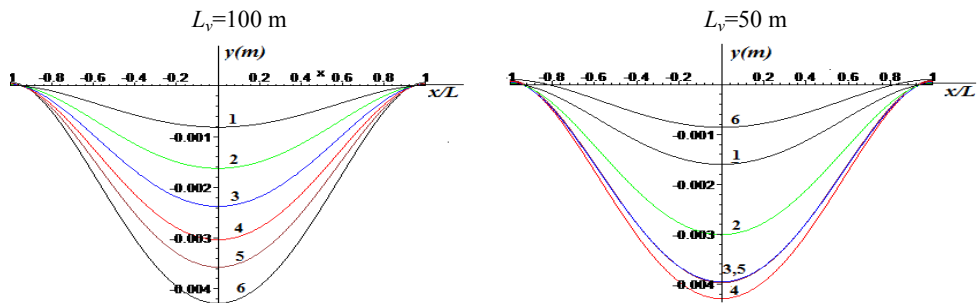


Fig. 5. Change in deflection along the length of the girder $x(m)$ (referred to the length of the girder L) for different values of time t (sec) and two wavelengths $L_v(m)$: 1 – $t = 0.02$, 2 – $t = 0.04$, 3 – $t = 0.06$, 4 – $t = 0.08$, 5 – $t = 0.1$, 6 – $t = 0.15$

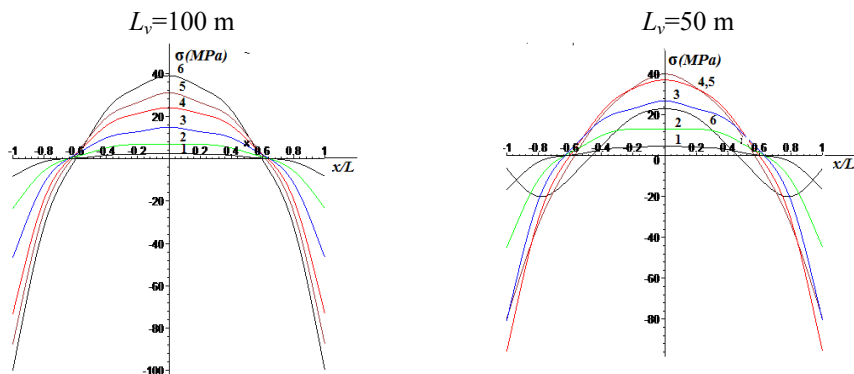


Fig. 6. Change in bending stresses σ (MPa) along the length of the girder x (m) (referred to the length of the girder L) for different values of time t (sec) and two wavelengths L_v (m): $1-t = 0.01$, $2-t = 0.03$, $3-t = 0.06$, $4-t = 0.1$, $5-t = 0.13$, $6-t = 0.19$

Comparison of the graphs presented in figure 4 indicates that long waves, in addition to changing the frequency composition of the oscillatory process, lead to a significant increase in the maximum values of stresses near the section where the girder is attached to the supports. From the analysis of the graphs (Figures 4 and 5) of deflections and stresses distribution along the length of the girder, it follows that during a certain period of time, the deflections and stresses reach their maximum values, and such states may appear periodically, due to the absence of wave-absorbing elements in the girder bridge structure.

4 Conclusions

1. A method of dynamic calculation of a girder bridge for the effect of longitudinal waves on the supports of girder bridges was proposed.
2. By calculation, the pattern of oscillatory processes in the girder sections was determined when the force effect is transferred to the girder bridge through the supports immersed in soil.
3. The influence of the frequency and amplitude of acting harmonic waves on the dynamic characteristics of the girder vibrations was estimated. It was established that the increase in the longitudinal wavelength acting on supports has practically no effect on the amplitudes of the girder deflections; it leads to a change in the frequency composition of the oscillatory process and an increase in the amplitude of stresses induced by the bending moments near the connection sections of the girder and the supports.
4. The reasons for the appearance of irreversible strains at the connections of bridges with supports, leading to a decrease in the quality indicators of the operational properties of bridges, were revealed.

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